Multi-target Preimage search using parallel Grover

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Introduction

Preimage

Let H be a function that $H : \{0,1\}^b \to \{0,1\}^b$. Preimage search is given an output y, find a x such that $H(x) = y$.

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It is desirable that given an output it should be computationally infeasible to find any input that maps to that output.

Introduction

Brute-force search for one preimage

Let H be a function that $H : \{0,1\}^b \rightarrow \{0,1\}^b$.
The brute force is to check every input x given an output y. The time complexity will be 2^b guesses using classical computers.
If we apply Grover's algorithm, using a quantum computer, the complexity decreases to $2^{b/2}$ guesses.

Introduction

Brute-force search for multi target preimages

Let H be a function that $H : \{0,1\}^b \to \{0,1\}^b$.
Now, we have a set of output y's, i.e., $Y = \{y_1, y_2, \ldots, y_t\}$ and we want to find one y_i and we verify every input x with a set of output Y.
If we **ignore several costs**, the complexity decreases to $2^b/t$ guesses in a classical computer.
If we apply Grover's algorithm, using a quantum computer, the complexity decreases to $2^{b/2}/t^{1/2}$ guesses.

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- Classical computer:
  - Single target: 2^b
  - Multi target: $t \cdot (2^b)/t$
- Quantum computer:
  - Single target: $2^{b/2}$
  - Multi target: $t \cdot (2^{b/2})/t^{1/2}$

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Parallel multi-target image attack for AES:

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However, it is pre-quantum.

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NIST has claimed that AES-128 is secure enough.

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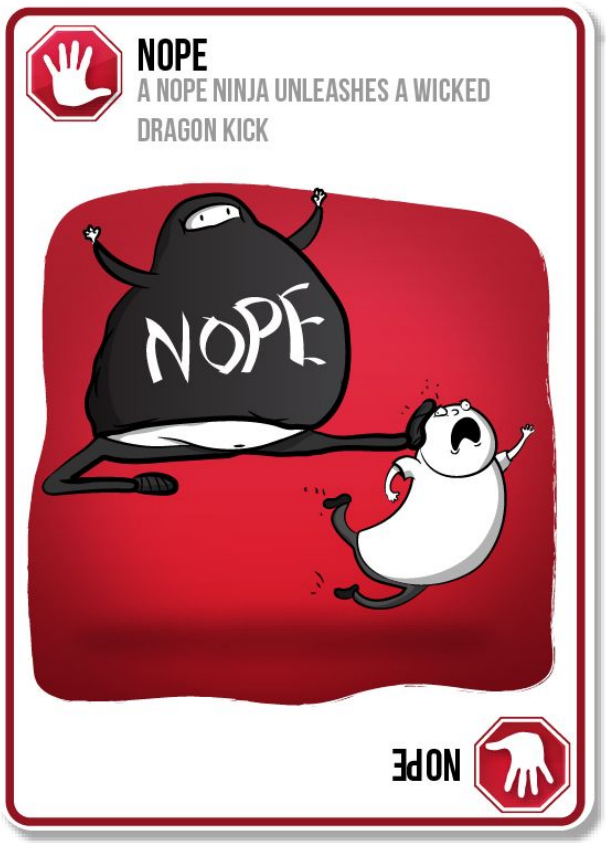


Introduction - Parallel rho method

Distinguished Point

Consider $H : \{0,1\}^b \to \{0,1\}^b$
Take x an input of H, $x' = H(x)$.
Thereafter, take x' and apply H again, $x'' = H(x')$.
It is possible to do it n times and we denote as $H^n(x)$.

Introduction - Parallel rho method

Distinguished Point

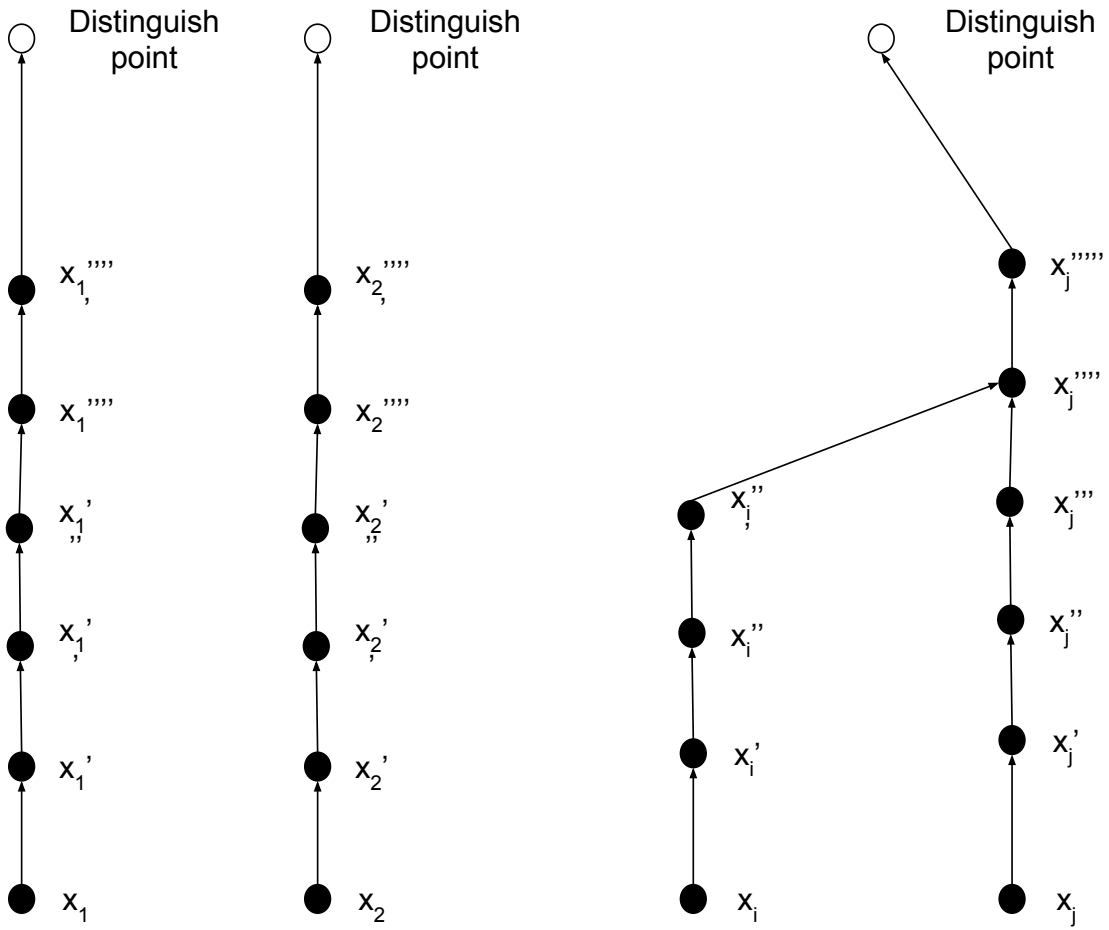
Consider $H : \{0,1\}^b \to \{0,1\}^b$

We want that our distinguished point satisfied $d = b/2$ and we denote as:

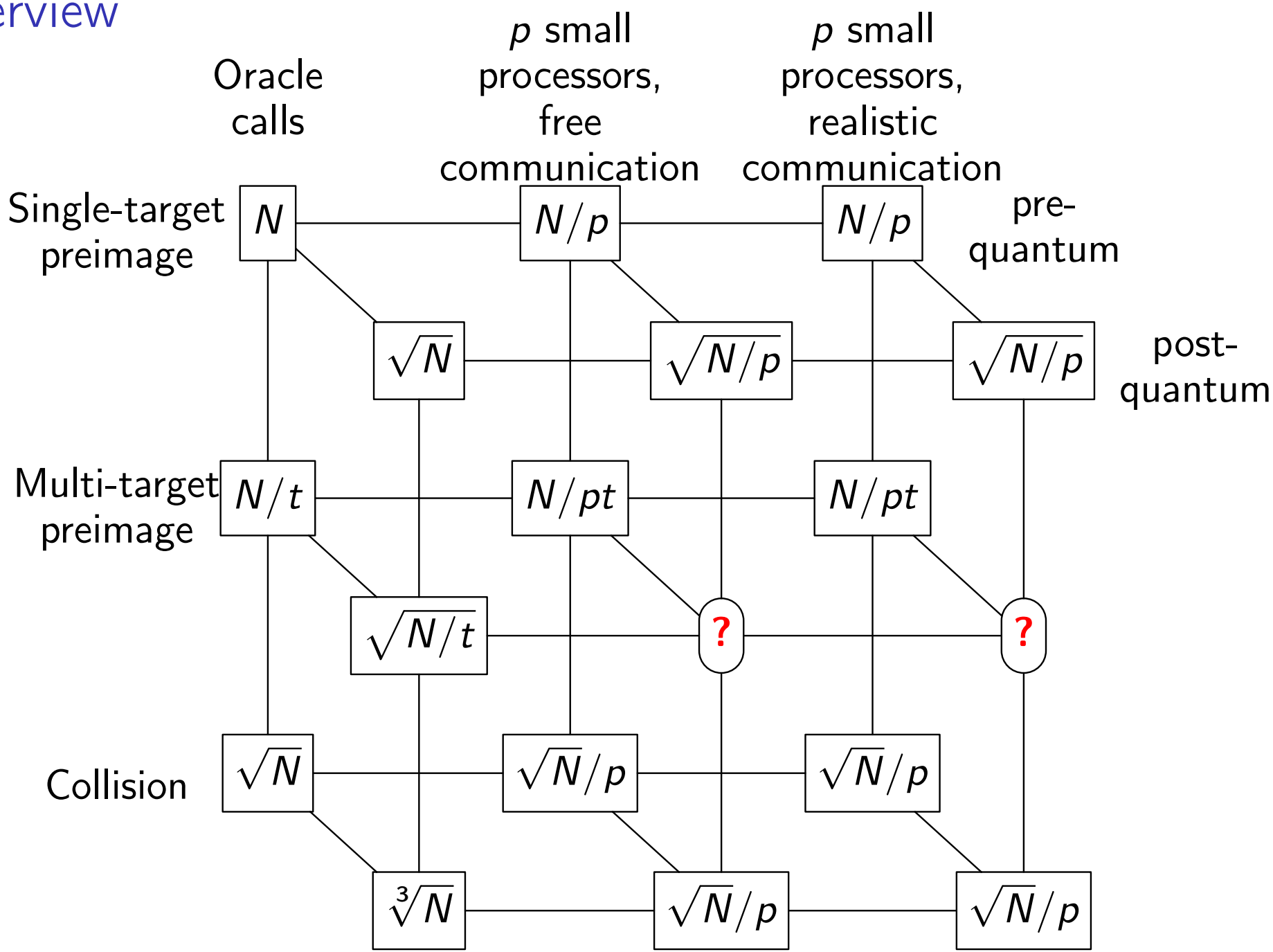
$$H_d(x) = \underbrace{0 \ldots 0}_{d \text{ zeros}} \{0,1\}^{b/2}$$

Introduction - Parallel rho method

Distinguished Point



Overview


Oracle calls
p small processors, free communication
p small processors, realistic communication
Single-target preimage
Multi-target preimage
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pre-quantum
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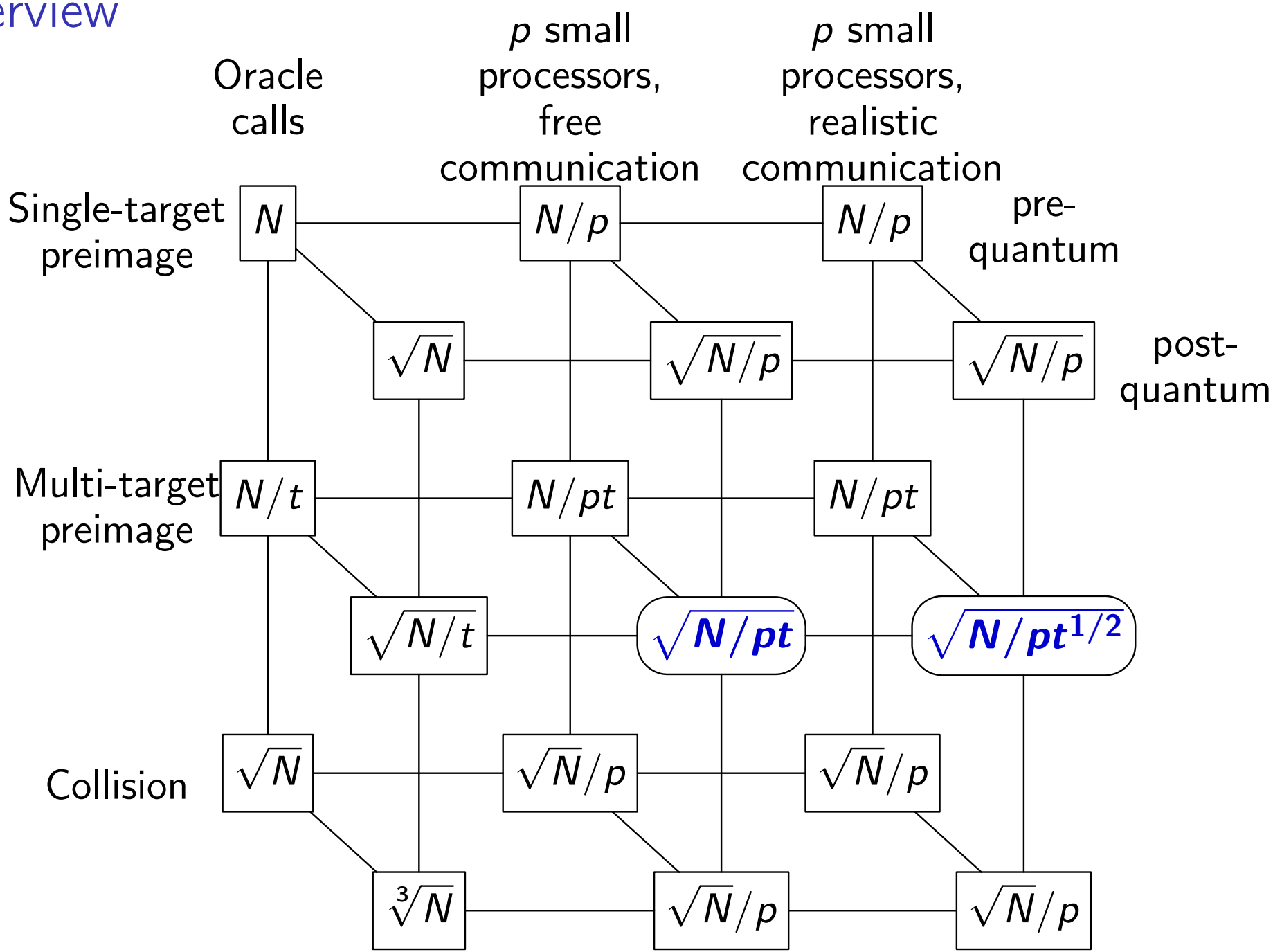
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Distinguished point in quantum setting

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- It is not possible to design a "simple circuit" for distinguished point;

- The sorting needs to be reversible too.

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Using classical computers

Example to compute $H^3(x)$:

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time 0:	x	0	0

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time 3:	x	$H^3(x)$	$H(x)$	$H^2(x)$
time 4:	x	$H^3(x)$	$H(x)$	0

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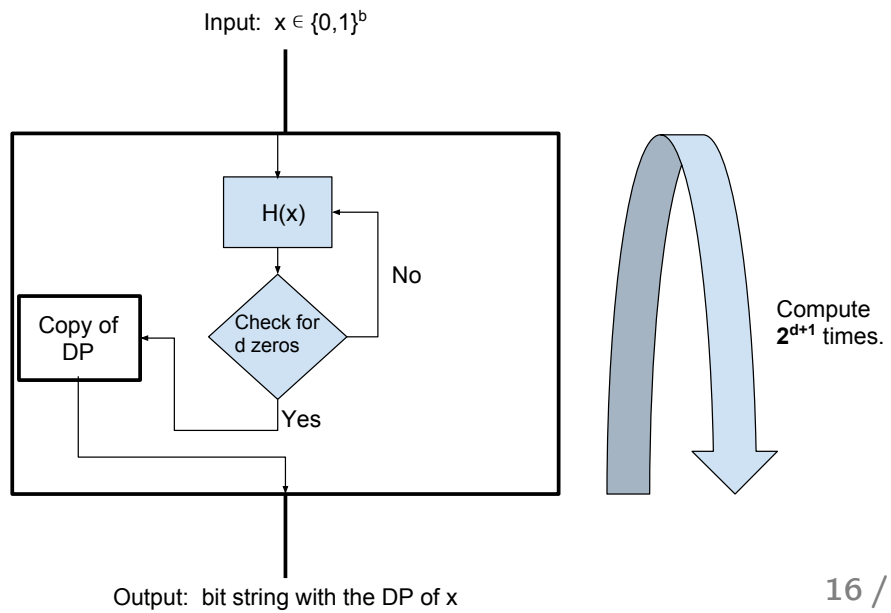
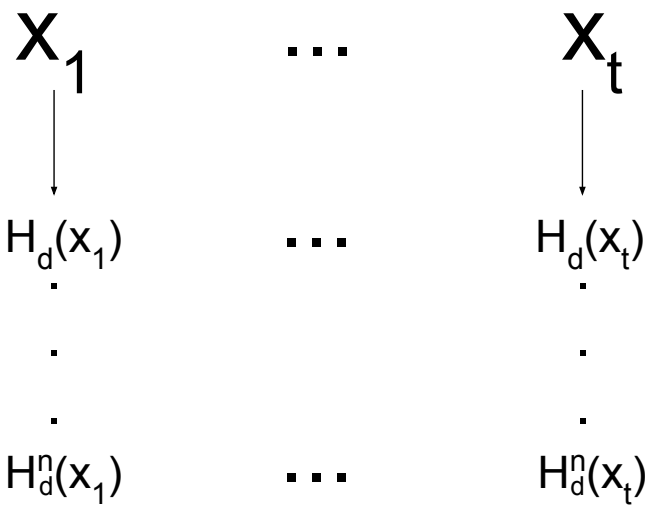
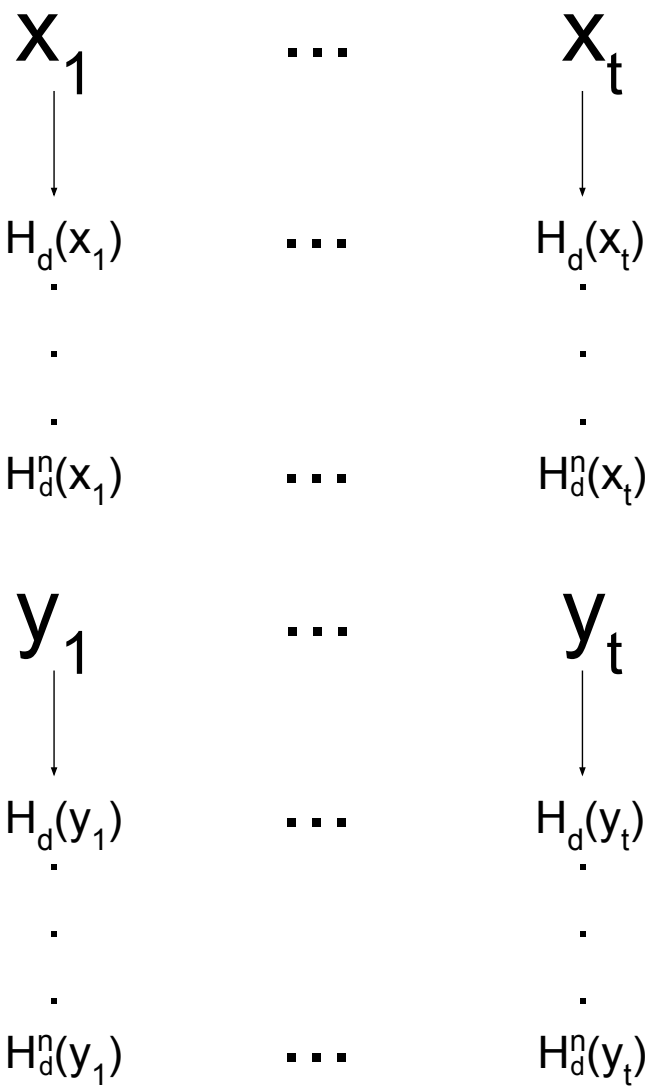
time 2:	x	0	$H(x)$	$H^2(x)$
time 3:	x	$H^3(x)$	$H(x)$	$H^2(x)$
time 4:	x	$H^3(x)$	$H(x)$	0
time 5:	x	$H^3(x)$	0	0

$H_d^n(x) =$




x_1 ... x_t
$H_d(x_1)$ ... $H_d(x_t)$
$H_d^n(x_1)$ ... $H_d^n(x_t)$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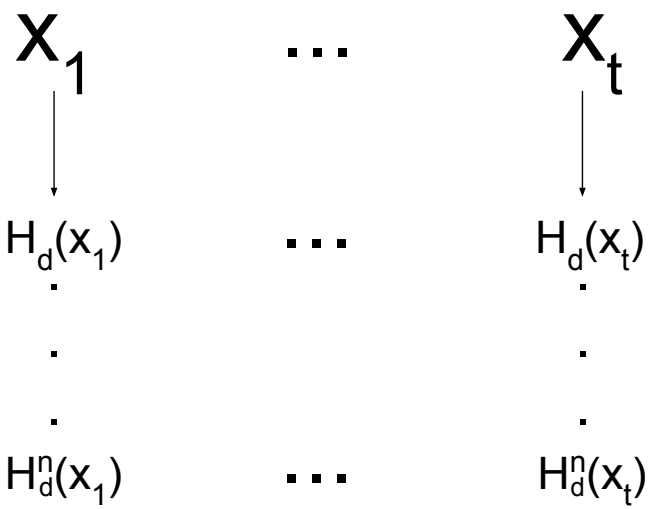

$$H_d^n(y_i) \stackrel{?}{=} H_d^n(x_j)$$

If this condition is true then we need to run classically:

$$H^{n_k}(x_i) = y_j$$

Reversibility

Reversibility of Distinguished point

- Bennett-Tompa technique to build a reversible circuit for H_d^n;
- It is possible to achieve $a + O(b \log_2 n)$ ancillas and gate depth $O(gn^{1+\epsilon})$.

[3] Efficient distributed quantum computing
Beals, Robert and Brierley, Stephen and Gray, Oliver and Harrow, Aram W. and Kutin, Samuel and Linden, Noah and Shepherd, Dan and Stather, Mark

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Reversibility of sorting on a mesh network

- Using the sorting strategy from "Efficient distributed quantum computing"[3];
- We used odd-even mergesort;
- It is possible to perform the sorting of t elements using $O(t(b + (\log t)^2))$ ancillas and $O(t^{1/2}(\log t)^2)$ steps.

---

[3] Efficient distributed quantum computing
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Fix images $y_1, \ldots, y_t$. We build a reversible circuit that performs the following operations:

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- Output 0 if a preimage was found, otherwise 1.

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- $\sqrt{2^{128}/2^{40}(2^{40})^{3/2}} \approx \sqrt{2^{128}/2^{100}}$
  $= \sqrt{2^{28}} = 2^{14}$ iterations.

Conclusion

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- Circuit uses $O(a + tb + t(\log t)^2)$ ancillas;
- Depth of $O(\sqrt{N/pt^{1/2}}(gt^{\epsilon/2} + (\log t)^2 \log b))$;
- Approximately $\sqrt{N/pt^{3/2}}$ iterations.
- Create the circuit using quantum simulator for AES;
  - We already implemented using libquantum; One round of AES with $11,100$ gates;

[4] (libquantum instead of LiQ$Ui\,|\rangle$);

[4] Applying Grover's algorithm to AES: quantum resource estimates Grassl, Markus and Langenberg, Brandon and Roetteler, Martin and Steinwandt, Rainer

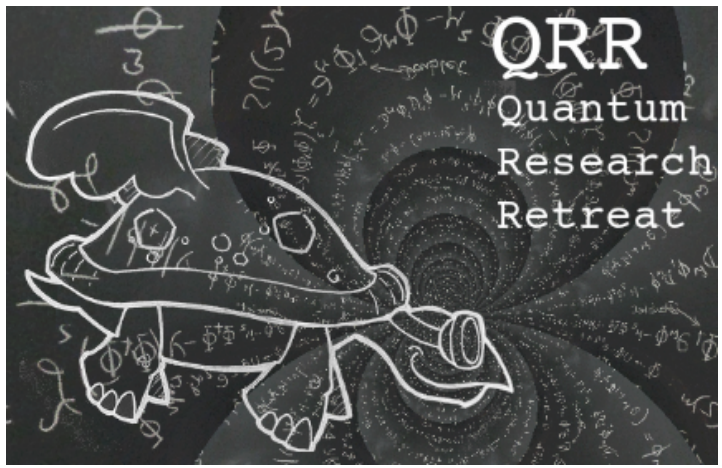
Outreach

- This work was at SAC 2017;
- We gave a talk at CWG (Crypto working group) reaching the Dutch community;
- We gave a talk at Quantum Cryptanalysis Seminar in Dagstuhl (Reaching the scientific community);
- Ei/ψ - Security in times of surveillance (General Public);

What's next?

- Check for the real number of qubits/gates giving an implementation;
- Change libquantum for "Big Integer";
- Implement the work from "Quantum resources estimates for ECC"[5];
- Finish the side channel attacks on ECC (work with Riscure);

- Quantum Research Retreat (QRR) in Eindhoven (mid December, https://cryptme.in/events/);



[5] Quantum resource estimates for computing elliptic curve discrete logarithms
Martin Roetteler, Michael Naehrig, Krysta M. Svore, Kristin Lauter

Questions

Thank you for your attention.
Questions?

gustavo@cryptme.in